

Lecture 14

Fundamentals of Business Mathematics

MATH-114 • BBA 2k26

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Learning Objectives

By the end of this lecture, students should be able to:

- calculate the compound amount of a single deposit;
- determine the present value of a future sum;
- apply compound-growth models to business and population contexts;
- distinguish a nominal annual rate from an effective annual rate;
- choose the appropriate formula and interpret the result.



From Lecture 13 to Lecture 14

In Lecture 13, we studied how money grows when interest is added to the balance.

Today we ask four related questions:

1. **Future value:** What will a deposit grow to?
2. **Present value:** How much must be invested today to reach a future goal?
3. **Applications:** How can the same growth model describe other quantities?
4. **Effective rate:** What annual rate is actually earned after compounding?



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Key idea

A single-payment computation involves one amount invested now or one amount received in the future.



Notation and the Compound-Amount Formula

Let

$$A = P\left(1 + \frac{i}{m}\right)^{mt}$$

where

- P = principal or amount invested today;
- A = compound amount after t years;
- i = nominal annual interest rate as a decimal;
- m = number of compounding periods per year;
- t = time in years.



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- m = number of compounding periods per year;
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Before substituting

Convert the percentage to a decimal and match the rate and time units. For example, quarterly compounding means $m = 4$.

Example 1: Find the Compound Amount

Question

A business deposits \$15,000 in an account paying 8% nominal annual interest, compounded quarterly. How much will be in the account after 6 years? How much interest will have been earned?



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A business deposits \$15,000 in an account paying 8% nominal annual interest, compounded quarterly. How much will be in the account after 6 years? How much interest will have been earned?

Plan

- $P = 15{,}000$, $i = 0.08$, $m = 4$, $t = 6$;
- quarterly rate $= 0.08/4 = 0.02$;
- number of periods $= 4(6) = 24$.



Example 1: Solution

$$A = 15{,}000\left(1 + \frac{0.08}{4}\right)^{4(6)}$$

$$A = 15{,}000(1.02)^{24} \approx \$24{,}126.56$$



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Interest earned:

$$I = A - P = 24{,}126.56 - 15{,}000 = \$9{,}126.56$$

Interpretation

After 6 years, the account balance is approximately \$24,126.56, including \$9,126.56 in interest.



A Reliable Method for Compound-Amount Questions

1. Identify the amount invested today P.
2. Write the nominal annual rate i as a decimal.
3. Identify compounding frequency m.
4. Convert the time to years t.
5. Calculate the periodic rate i/m and number of periods mt.
6. Substitute into the formula and round money to the nearest cent.



Present Value: Thinking Backward

Suppose a business needs a specified amount at a future date. Present value asks:

How much must be invested today to reach that future amount?

Starting with

$$A = P\left(1 + \frac{i}{m}\right)^{mt},$$

solve for P:

$$\boxed{P = \frac{A}{\left(1 + \frac{i}{m}\right)^{mt}}}$$



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- A is the future amount required.
- P is the equivalent amount today.
- The future amount is discounted using the stated interest rate.



Example 2: Present Value

Question

A company wants to have \$50,000 available in 3 years. How much should it invest today in an account paying 9% nominal annual interest, compounded monthly?



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Set up

$$A = 50{,}000, \quad i = 0.09, \quad m = 12, \quad t = 3$$

Use the present-value formula:

$$P = \frac{A}{(1 + i/m)^{mt}}$$



Example 2: Solution

$$P = \frac{50{,}000}{\left(1 + \frac{0.09}{12}\right)^{12(3)}}$$

$$P = \frac{50{,}000}{(1.0075)^{36}} \approx \$38{,}207.45$$



Example 2: Solution

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$$P = \frac{50{,}000}{(1.0075)^{36}} \approx \$38{,}207.45$$

Interpretation

At the stated rate, an investment of approximately \$38,207.45 today will grow to \$50,000 in 3 years.

The present value is less than the future amount because the investment earns interest over time.



Compound Amount and Present Value: Compare

	Compound amount	**Present value**
Known	Amount today P	Future amount A
Find	Future amount A	Amount today P
Formula	$A = P(1 + i/m)^{mt}$	$P = A/(1 + i/m)^{mt}$
Direction	Move forward in time	Discount backward in time



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Known	Amount today P	Future amount A
Find	Future amount A	Amount today P
Formula	$A = P(1 + i/m)^{mt}$	$P = A/(1 + i/m)^{mt}$
Direction	Move forward in time	Discount backward in time

Decision cue

If the question asks "how much will it grow to?" use compound amount. If it asks "how much is needed today?" use present value.



Other Applications: Growth Over Time

The same compound-growth structure can model a quantity that grows by a constant percentage each period:

$$Q_t = Q_0(1 + r)^t$$

where Q_0 is the initial quantity and r is the growth rate per period.

Possible business contexts include:

- sales or book sales increasing by a fixed annual percentage;
- student enrolment growing at a steady rate;
- prices increasing at a stated average rate;
- an investment growing under a specified compounding convention.



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- student enrolment growing at a steady rate;
- prices increasing at a stated average rate;
- an investment growing under a specified compounding convention.

Important: use the model only when the problem states or assumes a constant percentage growth rate.



Example 3: Student Enrolment Growth

Question

A university currently has 15,000 students. If enrolment grows by 3.5% per year and the rate remains constant, estimate the enrolment after 10 years.



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A university currently has 15,000 students. If enrolment grows by 3.5% per year and the rate remains constant, estimate the enrolment after 10 years.

Identify

$$Q_0 = 15{,}000, \quad r = 0.035, \quad t = 10$$

Use the annual growth model:

$$Q_t = Q_0(1 + r)^t$$



Example 3: Solution

$$Q_{10} = 15{,}000(1.035)^{10}$$

$$Q_{10} \approx 21{,}158.98$$



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$$Q_{10} \approx 21{,}158.98$$

Interpretation

The model predicts approximately 21,159 students after 10 years.

This is a mathematical estimate, not a guarantee: actual enrolment may change if the growth rate changes.



Example 4: Price Growth

Question

A particular model of laptop costs \$14,500 today. If its price increases at an average rate of 6% per year, compounded quarterly, what would the model predict its price to be in 5 years?



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Question

A particular model of laptop costs \$14,500 today. If its price increases at an average rate of 6% per year, compounded quarterly, what would the model predict its price to be in 5 years?

Set up

$$P = 14{,}500, \quad i = 0.06, \quad m = 4, \quad t = 5$$

$$A = P\left(1 + \frac{i}{m}\right)^{mt}$$



Example 4: Solution

$$A = 14{,}500\left(1 + \frac{0.06}{4}\right)^{4(5)}$$

$$A = 14{,}500(1.015)^{20} \approx \$19{,}529.40$$



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Interpretation

Under the stated model, the price after 5 years is approximately \$19,529.40.

Notice that the formula is the same as the compound-amount formula. Only the meaning of the quantity has changed.



Effective Annual Interest Rate

A nominal annual rate does not by itself show the total percentage earned in one year when interest is compounded more than once.

If the nominal annual rate is i and interest is compounded m times per year, the effective annual rate r_{eff} is

$$r_{\text{eff}} = \left(1 + \frac{i}{m}\right)^m - 1$$



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If the nominal annual rate is i and interest is compounded m times per year, the effective annual rate r_{eff} is

$$r_{\text{eff}} = \left(1 + \frac{i}{m}\right)^m - 1$$

- The nominal rate is the stated annual rate.
- The effective annual rate includes the effect of compounding within the year.
- For a positive nominal rate, more frequent compounding generally produces a higher effective annual rate.



Example 5: Effective Annual Rate

Question

An investment advertises a nominal annual interest rate of 12%, compounded quarterly. Determine its effective annual interest rate.



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An investment advertises a nominal annual interest rate of 12%, compounded quarterly. Determine its effective annual interest rate.

Identify

$$i = 0.12, \qquad m = 4$$

$$r_{\text{eff}} = \left(1 + \frac{i}{m}\right)^{m} - 1$$



Example 5: Solution

$$r_{\text{eff}} = \left(1 + \frac{0.12}{4}\right)^4 - 1$$

$$= (1.03)^4 - 1$$

$$= 1.12550881 - 1 = 0.12550881$$



Example 5: Solution

$$r_{\text{eff}} = \left(1 + \frac{0.12}{4}\right)^4 - 1$$

$$= (1.03)^4 - 1$$

$$= 1.12550881 - 1 = 0.12550881$$

Convert to a percentage:

$$r_{\text{eff}} \approx 12.551\%$$

Interpretation

A nominal rate of 12% compounded quarterly is equivalent to an effective annual rate of approximately 12.551%.



Compare Effective Rates

Suppose two accounts advertise the same nominal annual rate of 12%.

Compounding frequency	Formula	Effective annual rate
Annually	$(1+0.12)^1-1$	12.000%
Semiannually	$(1+0.12/2)^2-1$	12.360%
Quarterly	$(1+0.12/4)^4-1$	12.551%
Monthly	$(1+0.12/12)^{12}-1$	12.683%



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Monthly	$(1+0.12/12)^{12}-1$	12.683%

Takeaway

When comparing accounts with different compounding frequencies, compare their effective annual rates rather than only their nominal rates.



Choosing the Correct Formula

Question wording	Use
How much will the investment grow to?	$A = P(1 + i/m)^{mt}$
How much must be invested today?	$P = A/(1 + i/m)^{mt}$
What is the effective annual rate?	$r_{\text{eff}} = (1 + i/m)^m - 1$
How much will a quantity grow at $r\%$ per year?	$Q_t = Q_0(1 + r)^t$



Choosing the Correct Formula

Question wording	Use
How much will the investment grow to?	$A = P(1 + i/m)^{mt}$
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What is the effective annual rate?	$r_{\text{eff}} = (1 + i/m)^m - 1$
How much will a quantity grow at $r\%$ per year?	$Q_t = Q_0(1 + r)^t$

Always state what the final number represents: a future amount, a present value, a rate, or a projected quantity.



Suggested Textbook Problems: Section 8.2

Section 8.2 Follow-up Exercises *Textbook pp. 317–319*

- **Exercises 1–8:** Compound amount and interest earned.
- **Exercises 9–14:** Compound-growth applications and projections.
- **Exercises 15–20:** Present value; amount to deposit today to reach a future goal.
- **Exercises 21–24:** Determine the time required for an investment to reach a target.
- **Exercises 25–27:** Effective annual interest rates.
- **Exercises 28–32:** Determine the nominal annual rate needed to reach a target amount.



Additional Practice Problems

Try each problem before checking the solutions on the following slides.

1. **Compound amount:** A business invests \$25,000 at a nominal annual rate of 7.5%, compounded monthly. Find the amount after 4 years and the interest earned.



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1. **Compound amount:** A business invests \$25,000 at a nominal annual rate of 7.5%, compounded monthly. Find the amount after 4 years and the interest earned.
2. **Present value:** How much must be invested today to have \$100,000 in 3 years if the account pays 8% per year compounded quarterly?



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3. **Effective rate:** Find the effective annual rate for a nominal rate of 10% compounded monthly.



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3. **Effective rate:** Find the effective annual rate for a nominal rate of 10% compounded monthly.
4. **Single-payment application:** What amount should be invested today at 6% annual interest, compounded annually, to have \$40,000 after 8 years?



Additional Practice: Solutions (1–2)

1. Compound amount

$$A = 25{,}000\left(1 + \frac{0.075}{12}\right)^{12(4)}$$

$$A \approx \$33{,}714.98$$

$$I = A - P = 33{,}714.98 - 25{,}000 = \$8{,}714.98$$



Additional Practice: Solutions (1–2)

1. Compound amount

$$A = 25{,}000\left(1 + \frac{0.075}{12}\right)^{12(4)}$$

$$A \approx \$33{,}714.98$$

$$I = A - P = 33{,}714.98 - 25{,}000 = \$8{,}714.98$$

2. Present value

$$P = \frac{100{,}000}{\left(1 + \frac{0.08}{4}\right)^{4(3)}}$$

$$P \approx \$78{,}849.32$$



Additional Practice: Solutions (3–4)

3. Effective annual rate

$$r_{\text{eff}} = \left(1 + \frac{0.10}{12}\right)^{12} - 1 \approx 0.104713$$

$$\boxed{r_{\text{eff}} \approx 10.471\%}$$



Additional Practice: Solutions (3–4)

3. Effective annual rate

$$r_{\text{eff}} = \left(1 + \frac{0.10}{12}\right)^{12} - 1 \approx 0.104713$$

$$\boxed{r_{\text{eff}} \approx 10.471\%}$$

4. Present value

$$P = \frac{40{,}000}{(1.06)^8} \approx \$25{,}096.49$$

