

Lecture 17

Fundamentals of Business Mathematics

MATH-114 • BBA 2k26

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Lecture 17 Outline: Introduction to Matrices

- 9.1: Matrix notation and applications
- 9.2: Vectors, square matrices, identity matrices and transpose
- 9.3: Addition, subtraction and scalar multiplication



9.1: Why Use Matrices?

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S_2	15	40	25

$$A = \begin{pmatrix} 20 & 35 & 10 \\ 15 & 40 & 25 \end{pmatrix}.$$



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$$A = \begin{pmatrix} 20 & 35 & 10 \\ 15 & 40 & 25 \end{pmatrix}.$$

Rows identify stores; columns identify products. Keeping this ordering consistent makes later calculations meaningful.



Matrix Notation and Dimension

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}.$$



Matrix Notation and Dimension

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- Dimension: $m \times n$ (rows first, columns second).
- a_{ij} denotes the element in row i, column j.
- A matrix of dimension $m \times n$ contains mn elements.



Reading a Matrix

Classroom check: For $A = \begin{pmatrix} 20 & 35 & 10 \\ 15 & 40 & 25 \end{pmatrix}$, determine its dimension, a_{12} and a_{23}.



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The entry a_{23} represents sales of product 3 at store 2.



9.2: Vectors

A row vector has one row; a column vector has one column.

$$u = \begin{pmatrix} 4 & 7 & 2 \end{pmatrix}, \qquad v = \begin{pmatrix} 4 \\ 7 \\ 2 \end{pmatrix}.$$



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$$u = \begin{pmatrix} 4 & 7 & 2 \end{pmatrix}, \qquad v = \begin{pmatrix} 4 \\ 7 \\ 2 \end{pmatrix}.$$

Their dimensions are 1×3 and 3×1, respectively. A row vector and a column vector may contain the same numbers but have different dimensions.



Square and Identity Matrices

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Its primary diagonal contains 2 and 4. An identity matrix is square, with ones on the primary diagonal and zeros elsewhere.

$$I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$



Transpose of a Matrix

For an $m \times n$ matrix A, its transpose A^T has dimension $n \times m$.

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Rows of A become columns of A^T.

$$\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}^T = \begin{pmatrix} a & d \\ b & e \\ c & f \end{pmatrix}.$$



Example 1: Transpose

To find the transpose of the matrix

$$A = \begin{pmatrix} 3 & 2 \\ 4 & 0 \\ 1 & -2 \end{pmatrix}$$



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Solution: A is 3×2, so A^T is 2×3.

$$\boxed{A^T = \begin{pmatrix} 3 & 4 & 1 \\ 2 & 0 & -2 \end{pmatrix}}.$$



Example 2: Transpose of a Square Matrix

Let's apply this logic in finding the transpose of

$$B = \begin{pmatrix} 3 & 0 & 6 \\ 5 & 1 & 3 \\ 2 & -1 & 4 \end{pmatrix}.$$



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Let's apply this logic in finding the transpose of

$$B = \begin{pmatrix} 3 & 0 & 6 \\ 5 & 1 & 3 \\ 2 & -1 & 4 \end{pmatrix}.$$

Solution: Read each row as a column.

$$\boxed{B^T = \begin{pmatrix} 3 & 5 & 2 \\ 0 & 1 & -1 \\ 6 & 3 & 4 \end{pmatrix}}.$$



9.3: Matrix Addition and Subtraction

Compatibility

Two matrices may be added or subtracted only if their dimensions are identical.



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Compatibility

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If A and B are $m \times n$, then

$$(A + B)_{ij} = a_{ij} + b_{ij}, \qquad (A - B)_{ij} = a_{ij} - b_{ij}.$$

The result has dimension $m \times n$.



Example 3: Matrix Addition

Given

$$A = \begin{pmatrix} 1 & 3 \\ 4 & -2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -3 & 2 \\ 0 & 4 \end{pmatrix}$$

Find $A + B$.



Example 3: Matrix Addition

Given

$$A = \begin{pmatrix} 1 & 3 \\ 4 & -2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -3 & 2 \\ 0 & 4 \end{pmatrix}$$

Find $A + B$.

$$A + B = \begin{pmatrix} 1 + (-3) & 3 + 2 \\ 4 + 0 & -2 + 4 \end{pmatrix} = \boxed{\begin{pmatrix} -2 & 5 \\ 4 & 2 \end{pmatrix}}.$$



Example 4: Matrix Subtraction

Using the same matrices,

$$A = \begin{pmatrix} 1 & 3 \\ 4 & -2 \end{pmatrix}, \qquad B = \begin{pmatrix} -3 & 2 \\ 0 & 4 \end{pmatrix}.$$

Find $B - A$.



Example 4: Matrix Subtraction

Using the same matrices,

$$A = \begin{pmatrix} 1 & 3 \\ 4 & -2 \end{pmatrix}, \qquad B = \begin{pmatrix} -3 & 2 \\ 0 & 4 \end{pmatrix}.$$

Find $B - A$.

$$B - A = \begin{pmatrix} -3-1 & 2-3 \\ 0-4 & 4-(-2) \end{pmatrix} = \boxed{\begin{pmatrix} -4 & -1 \\ -4 & 6 \end{pmatrix}}.$$



Scalar Multiplication

A scalar is a real number. To multiply a matrix by a scalar k, multiply every element by k.

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$$3\begin{pmatrix} 2 & -1 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 6 & -3 \\ 0 & 12 \end{pmatrix}.$$



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$$3\begin{pmatrix} 2 & -1 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 6 & -3 \\ 0 & 12 \end{pmatrix}.$$

The dimension does not change. A scalar multiple is different from the product of two matrices.



Additional Practice Problems

Two branches report quarterly sales (units) for three products:

$$A = \begin{pmatrix} 20 & 30 & 40 \\ 10 & 25 & 35 \end{pmatrix}, \qquad B = \begin{pmatrix} 25 & 28 & 45 \\ 15 & 30 & 32 \end{pmatrix}.$$

1. State the dimension of A and interpret a_{23}.
2. Find A^T.
3. Find total sales $A + B$ and changes $B - A$.
4. If next quarter's forecast is twice the first quarter's sales, find $2A$.



Additional Practice: Solutions

$$\dim A = 2 \times 3, \quad a_{23} = 35 \text{ (branch 2, product 3).}$$



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$$A^T = \begin{pmatrix} 20 & 10 \\ 30 & 25 \\ 40 & 35 \end{pmatrix}.$$

$$A + B = \begin{pmatrix} 45 & 58 & 85 \\ 25 & 55 & 67 \end{pmatrix}, \quad B - A = \begin{pmatrix} 5 & -2 & 5 \\ 5 & 5 & -3 \end{pmatrix}.$$



Additional Practice: Solutions

$$\dim A = 2 \times 3, \quad a_{23} = 35 \text{ (branch 2, product 3)}.$$

$$A^T = \begin{pmatrix} 20 & 10 \\ 30 & 25 \\ 40 & 35 \end{pmatrix}.$$

$$A + B = \begin{pmatrix} 45 & 58 & 85 \\ 25 & 55 & 67 \end{pmatrix}, \quad B - A = \begin{pmatrix} 5 & -2 & 5 \\ 5 & 5 & -3 \end{pmatrix}.$$

$$2A = \begin{pmatrix} 40 & 60 & 80 \\ 20 & 50 & 70 \end{pmatrix}.$$



Suggested Textbook Problems

Section 9.2

- Problems 1–10: Dimensions and transposes.
- Problems 11–12: Constructing matrices from element rules.

Section 9.3 (first part)

- Problems 1–6: Addition, subtraction and scalar multiplication.

Section 9.1 has no separate numbered follow-up exercise set.

